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CAVITATING FLOWS

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I. Introduction

There are very few differences between the fluid dynamics of liquids and gases. The viscosity of water, for example, is 10^{-2} poise at 20° C while the viscosity of air at this temperature is about 2×10^{-4} poise. The kinematic viscosity of water is 10^{-2} cm^2/sec compared with 0.15 cm^2/sec for air. As one would expect from simple kinetic theory, the viscosity of gases increases with increasing temperature; the viscosity of liquids on the other hand decreases rather rapidly as the temperature rises. While the speed of sound in water is about four times that in air, there is a more interesting consequence of the equation of state. A pressure pulse with an intensity of several hundred psi, which propagates as a strong shock in air, will propagate acoustically in water with a negligible production of entropy.

These differences between liquid flows and gas flows which have just been enumerated cannot be considered essential. The difference which is basic arises from the fact that, in contrast with gases, liquids can support negative pressures, or tensions. When the tensions are sufficiently large, the liquid ruptures and the vapor phase appears. It is the growth and collapse of the vapor phase within a liquid which is the topic which will be considered here.

It might be expected that an uncontaminated liquid would withstand very large tensions before breaking. The appearance of a vapor cavity within a liquid is opposed by surface tension forces and the initial size of such a cavity is of the order of the distance between molecules. On this basis the tension required to break the liquid would be approximately $2\sigma/R$ where σ is the surface tension and R is the intermolecular distance. For water this tensile strength is

$$T \sim \frac{2 \times 70 \text{ dynes/cm}}{10^{-8}\text{cm}} \sim 10^{10} \frac{\text{dynes}}{\text{cm}^2} \cong 10^4 \text{ atm.}$$

The observed tensile strength of water even after the most careful preparation is two orders of magnitude smaller than this value. This decrease in observed tensile strength is ascribed to the nucleation of the liquid by energetic ionizing cosmic radiation. Samples of water, or other liquids, which have not been very carefully purified show very small tensile strengths so that they have an appreciable population of nuclei, or impurities, of microscopic, or submicroscopic, dimensions which serve as sites upon which the vapor phase may appear. The growth of a nucleus from microscopic initial size into a bubble of macroscopic dimensions is a problem of some complexity which is greatly simplified by supposing that the growth is spherically symmetric. An external condition which tends to disturb spherical symmetry is gravity. We shall neglect this source of asymmetry and this neglect implies that the bubble growth cannot be followed for so long a time that the bubble acquires an appreciable velocity of translation because of its buoyancy. This translational motion distorts an initially spherical cavity. Aside from such external sources of asymmetry bubble growth tends to follow the spherical shape (Ref. 1). When a bubble collapses, on the other hand, asymmetry effects (Ref. 1) appear toward the end of the collapse even in the absence of gravity or other external sources of asymmetry. These asymmetry effects are pronounced only toward the end of the collapse and will not be examined in detail here.

II. Growth of Vapor Bubbles

The bubble which is supposed to remain spherical has a boundary radius $\vec{R}(t)$. We let $\vec{r}$ be the position vector to any point in the liquid measured from the fixed center of the bubble and at this point we denote the velocity of the liquid by $\vec{v}(r, t)$. Then the equation of motion for $\vec{v}$ may be written as*

$$\rho\left\{\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right\} = -\nabla p + \left(\frac{4\mu}{3} + \zeta\right)\nabla(\nabla\cdot\vec{v}) - \mu\nabla\times(\nabla\times\vec{v}) \quad , \tag{1}$$

*See, for example, L. Landau and E. Lifshitz, Fluid Mechanics, §15.

where μ is the ordinary coefficient of viscosity and ζ is a second coefficient of viscosity related to the bulk viscosity. Since we have restricted the liquid motion to spherical symmetry, it follows that the motion is irrotational, $\nabla \times \vec{v} = 0$. The velocity field then has a potential

$$\vec{v} = - \nabla\varphi(r, t) \quad . \tag{2}$$

The momentum equation, because of the irrotationality of the motion, reduces to

$$\rho\left\{\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla\vec{v}\right\} = - \nabla p + \left(\frac{4\mu}{3} + \zeta\right) \nabla(\nabla \cdot \vec{v}) \quad . \tag{3}$$

It is evident that the last term in Eq. (3) disappears either if the viscosity vanishes or if the compressibility vanishes. The viscosity of a liquid such as water is small and the compressibility is also small so that the last term in Eq. (3) will ordinarily be unimportant. If then we use the general vector identity,

$$\vec{v} \cdot \nabla\vec{v} = \frac{1}{2} \nabla(v^2) - \vec{v} \times (\nabla \times \vec{v}) \quad , \tag{4}$$

we obtain

$$\frac{\partial \vec{v}}{\partial t} + \frac{1}{2} \nabla(v^2) = - \frac{1}{\rho} \nabla p \quad . \tag{5}$$

Equation (5) integrates to the generalized Bernoulli relation

$$- \frac{\partial \varphi}{\partial t} + \frac{v^2}{2} = - \int \frac{dp}{\rho} \quad . \tag{6}$$

As we shall see in the discussion of bubble growth, the fluid velocities are all moderate so that compressibility effects may be neglected. For bubble collapse, the liquid velocities are again small except near the end of the collapse. It follows for the collapse case also that compressibility effects are unimportant for most of the motion. If we neglect the compressibility of the liquid, we obtain from Eq. (6)

$$- \frac{\partial \varphi}{\partial t} + \frac{v^2}{2} + \frac{p(r, t)}{\rho} = \frac{P_o}{\rho} \tag{7}$$

where P_o is the pressure in the liquid at infinity which may be a function of the time.

The bubble boundary, $R(t)$, has a speed $dR/dt \equiv \dot{R}$. The equation of continuity, or the equation of conservation of mass, requires for spherically symmetric, incompressible flow that

$$\varphi(r,t) = \frac{\dot{R}R^2}{r} . \tag{8}$$

We now may evaluate Eq. (7) at $r = R$ and, with use of Eq. (8) obtain the equation of motion of the bubble boundary:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p(R)-P_o}{\rho} . \tag{9}$$

The general equations of motion in the liquid, and the equation of motion of the bubble boundary in particular, are based on the assumption first, that the motion is spherically symmetric and second, that the effects of compressibility may be neglected. Viscous effects have not been neglected but they appear only in the boundary condition at the bubble wall. With spherically symmetric motion the condition of stress continuity across the bubble boundary reduces to a condition on normal stress only. If p' is the pressure in the bubble and $p(R)$ is the pressure in the liquid at the bubble wall, then the boundary condition is

$$p' = p(R) + \frac{2\sigma}{R} + \frac{4\mu\dot{R}}{R} , \tag{10}$$

where σ is the surface tension and μ the coefficient of viscosity. If the pressure within the bubble is not uniform throughout the bubble, then p' must be the pressure in the gas phase at the bubble wall. The pressure in the gas may be taken to be uniform so long as the bubble wall velocity, $\dot{R}$, is small compared with the speed of sound in the gas phase. We shall make this assumption which is a more stringent requirement than the one already imposed in neglecting compressibility effects in the liquid since the speed of sound in a gas is less than the speed of sound in a liquid.

The problem of the motion of a bubble may now be solved by means of Eq. (9) with the boundary condition (10) within the restrictions which we have imposed.

In a liquid which contains dissolved gas we may suppose that a nucleus from which a bubble grows will contain this gas. As the bubble grows, however, the contribution of the permanent gas quickly becomes negligible and the pressure within the bubble is essentially the vapor pressure of the liquid. The diffusion of additional dissolved permanent gas into the bubble is too slow a process to contribute appreciably to a single expansion process. We shall consider, therefore, the growth of a vapor bubble in a liquid under tension and shall exhibit the important details by taking two limiting conditions which are of physical importance. The one condition corresponds to what is usually called cavitation, and the other is familiar as "boiling". As a vapor bubble expands, the latent heat of evaporation required for the increased mass of vapor must be supplied from the liquid at the bubble wall. The density of a vapor is a decreasing function with decreasing temperature so that at low temperature the increased mass of vapor required to fill the expanding bubble is small and the latent heat requirement is correspondingly small. This is the subcooled liquid situation and the bubble expansion is controlled as we shall see by the inertia of the liquid. At high temperatures, the vapor mass and the latent heat are both significant and the bubble growth is limited by the rate at which the heat flows into the bubble from the liquid. We shall call this situation the liquid boiling situation.

1. Bubble Growth in Subcooled Liquids - Cavitation. We shall consider the case of vapor bubble growth with a constant static pressure P_o at infinity. The liquid is under the tension

$$(P_o - p_v) < 0$$

where p_v is the vapor pressure. The subcooled case from the point of view of the dynamics is characterized by the fact that p_v is very nearly constant throughout the bubble growth. A numerical example may be used to show the essential features of this cavitation case. Suppose that a vapor bubble grows from microscopic size to a radius say, $R = 0.1$ cm in water at 15°C. A characteristic time for such a growth under moderate tension is about $\tau = 10^{-3}$ sec. The total mass of vapor which is evaporated into the bubble is $(4\pi/3)R^3\rho'$ where ρ' is the vapor density. If L is the latent heat of evaporation per gram, than the total heat required is

$$Q = (4\pi/3)R^3\rho' L \quad .$$

Now at 15°C, ρ' is approximately 0.13×10^{-4} gm/cm^3, and $Q = 3.1 \times 10^{-5}$ calories. This heat is taken out of a water layer surrounding the bubble. The effective thickness of this layer is determined by the thermal diffusivity of water, D,

$$D = \frac{k}{\rho c} \quad ,$$

where k is the thermal conductivity, ρ is the density, and c is the specific heat of water. At 15°C, D has the value 1.4×10^{-3} cm^2/sec and the thickness of the water layer from which the heat comes is roughly given by

$$d \sim (D\tau)^{\frac{1}{2}}$$

and for $\tau = 10^{-3}$ sec, $d \sim 1.2 \times 10^{-3}$ cm. We may remark that the estimate of the thickness is not sensitive to our choice of the growth time τ. The volume of the water layer is of the order of magnitude $4\pi R^2 d$ and the corresponding mass of water is 1.5×10^{-4} gm. Finally the temperature drop of this water layer, ΔT, is given by

$$\Delta T \sim \frac{(4\pi/3)R^3 \rho' L}{4\pi R^2 d\rho c} = \frac{R}{3d} \frac{\rho'}{\rho} \frac{L}{c} \quad .$$

This expression gives $\Delta T \sim 0.2$°C for the cooling of the water at the bubble boundary. The vapor pressure of water at 15°C is 1.7×10^4 dynes/cm^2. If water is cooled below 15°C by the amount ΔT given above, the decrease in the vapor pressure may be estimated to be

$$\Delta p_v \sim \left(\frac{dp_v}{dT}\right)_{15°C} \times \Delta T = 1.1 \times 10^3 \times 0.2 \sim 0.2 \times 10^3 \text{dynes/cm}^2 \quad .$$

This decrease in vapor pressure is clearly seen to be negligible compared with p_v.

We see that thermodynamic effects are unimportant in cavitation bubble growth. The vapor pressure is essentially unaffected by the bubble expansion so that when P_o is also constant, the tension of the liquid is constant. For this case we may see some further features of cavitation bubble growth. The general equation of motion (9) has the form

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p_v - 2\sigma/R - 4\mu\dot{R}/R - P_o}{\rho} ,$$

where we have used (10) with $p' = p_v(T)$, the vapor pressure of the liquid at temperature T. We suppose that the cavitation bubble grows from a nucleus at rest. It is then easy to see that the term $4\mu\dot{R}/R$ is always small and initially zero. We have initially that $2\sigma/R$ is roughly equal to $p_v - P_o$, and as R increases, we can see that this term soon becomes unimportant. Finally we may use the identity

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{2R^2\dot{R}}\frac{d}{dt}(R^3\dot{R}^2)$$

and then we find at once that

$$R^3\dot{R}^2 \cong \frac{2(p_v-P_o)}{\rho}\int^R R^2\dot{R}\,dt = \frac{2(p_v-P_o)}{\rho}\int^R R^2 dR ,$$

or

$$\dot{R} \cong \left[\frac{2}{3}\frac{(p_v-P_o)}{\rho}\right]^{\frac{1}{2}} . \tag{11}$$

Equation (11) gives the asymptotic growth speed, or the growth speed when the bubble has grown appreciably greater than its initial size. This growth is inertia-controlled, that is, it is determined entirely by the inertia of the liquid.

2. Bubble Growth in Superheated Liquids - - Boiling. In cavitating flows with "cold" water the tensions $p_v - P_o$ are usually large compared with p_v. These tensions are of course transient and the cavitation bubble when it comes into a region of positive pressure $P_o > p_v$ will collapse. Some features of this collapse will be considered below. In superheated liquids the situation of greatest interest corresponds to a positive ambient pressure P_o which is exceeded only slightly by p_v. We shall now see the consequences of this relationship by considering a specific example.

Let us suppose that the superheated liquid is water at an ambient pressure of 1 atmosphere. Further we suppose that the water is superheated to a few degrees over 100°C so that the vapor pressure exceeds the ambient pressure. Now at 100°C the density of water vapor is 6×10^{-4} gm/cm^3 which is 46 times its value of 15°C. The temperature drop is now not negligible being approximately 13°C instead of 0.2 °C. If the water is superheated to 105°C or 110°C such a large temperature drop would be impossible for a growing bubble since the vapor pressure for such a reduced temperature would be less than the ambient pressure. The growth time τ that we considered reasonable for a cavitation bubble is much too short for a boiling bubble. We might also note that the vapor pressure drop for a given drop in temperature ΔT in the neighborhood of 100°C is large compared with its value at 15°C. At 15°C we had $(dp_v/dt) = 1.1 \times 10^3 \text{dyne/cm}^2\,{}^\circ\text{C}$ while at 100°C$(dp_v/dt) = 38 \times 10^3 \text{dyne/cm}^2\,{}^\circ\text{C}$.

It is clear that the dynamics of the growth of a "boiling" bubble is quite different from that of a cavitation bubble. It is evident that the growth rate will be limited by the rate at which the heat required for evaporation can flow into the bubble. As a consequence the growth rates are much slower than those found previously. For moderate superheats the effects of liquid inertia are unimportant. The growth rate for a boiling bubble may be estimated in the following way. Suppose that the superheated liquid is at a temperature T_o. Also suppose that the "boiling temperature", that is the temperature at which the vapor pressure equals the ambient pressure P_o, is T_b. Then

$$p_v(T_o) > p_v(T_b) = P_o \quad .$$

Now the cooling effect will lower the temperature in the bubble so that it becomes closer to T_b as the bubble becomes larger. This temperature T_b is also near the temperature in the liquid at the bubble wall. The temperature in the liquid rises from its value at the bubble wall to the value T_o in a distance of the order of magnitude $(Dt)^{\frac{1}{2}}$ where D is the thermal diffusivity of the liquid. This distance is the diffusion length for heat flow and is familiar to us from the nature of the fundamental solutions of the heat flow equation. It follows that the heat flow into the bubble is given approximately by

$$\dot{Q} \cong 4\pi R^2 k \frac{(T_o - T_b)}{(Dt)^{\frac{1}{2}}} \quad , \tag{12}$$

where $R = R(t)$ is again the bubble radius and k is the thermal conductivity of the liquid. On the other hand we know that the heat requirement per unit time for evaporation is

$$\dot{Q} \cong L \frac{d}{dt}\left(\frac{4\pi}{3} R^3\rho'\right) \cong 4\pi R^2\dot{R}L\rho' \quad . \tag{13}$$

In Eq. (13) we may neglect a term which is proportional to $d\rho'/dt$. It is to be expected that the change in the vapor density with time is much less important than the rate of change of the bubble volume with time since we expect that the bubble temperature changes slowly and by a small amount in time. If now we equate these two values for $\dot{Q}$, we find the asymptotic growth speed for a boiling bubble in a superheated liquid:

$$\dot{R} \cong \frac{k}{L\rho'} \frac{(T_o - T_b)}{(Dt)^{\frac{1}{2}}} \quad .$$

An accurate calculation (Ref. 2) gives this result with an additional factor $(\pi/3)^{\frac{1}{2}}$.

In the dynamics of growth of the cavitation bubble and of the boiling bubble there are several common features. The inertial effects of the vapor are unimportant; viscosity is also unimportant as is the compressibility of the liquid. Surface tension is unimportant except for very early stages of growth. We may expect that some of these neglects will not be possible when we consider the collapse of a vapor bubble.

III. Collapse of Vapor Bubbles

We now consider some features of the collapse of a vapor bubble. We suppose that the ambient pressure P_o exceeds the vapor pressure p_v. The equation of motion in the incompressible approximation which we have already used is

$$\frac{1}{2R^2\dot{R}} \frac{d}{dt} (R^3\dot{R}^2) = \frac{p_v - P_o - 2\sigma/R}{\rho} \tag{14}$$

where we have for simplicity dropped the viscous term, a simplification which is justified so long as $\mu\dot{R} \ll \sigma$. Let us suppose that at $t = 0$ the bubble is at rest with radius R_o. Then Eq. (14) upon integration gives

$$\dot{R}^2 = \frac{2(P_o - p_v)}{3\rho}\left(\frac{R_o^3}{R^3} - 1\right) + \frac{2\sigma}{\rho R}\left[\left(\frac{R_o}{R}\right)^2 - 1\right] \quad . \tag{15}$$

It has, of course, been assumed that $(P_o - p_v)$ is constant. Equation (15) shows that the collapse velocity approaches infinity like $R^{-3/2}$ as the radius R approaches zero. It is now clear that many of the simplifying assumptions which have been made cannot remain valid in the collapse motion.

In the collapse motion a stage will be reached in which the heat of condensation will produce a temperature rise so that the vapor pressure will increase. It may be shown by numerical treatment that the dynamics of the collapse is not greatly altered by the temperature rise in a compressed bubble. An effect which is much more significant for cavitation bubbles than the thermal effects is that of compressibility. The liquid compressibility becomes significant at an earlier stage in the collapse history and also has a greater effect on the bubble dynamics. The problem of the collapse of a spherical bubble in a compressible liquid has received considerable attention. An acoustic correction to the incompressible solution, that is, a solution valid to the first power of $|\dot{R}|/c$ where c is the sound speed, was developed by Herring (Ref. 3,4). A somewhat simpler deduction of this acoustic approximation to the compressible flow was obtained by Trilling (Ref. 5). As $|\dot{R}|$ continues to increase as R decreases, the acoustic theory soon becomes inadequate. A very successful approximation to the compressible flow equations was obtained by Gilmore (Ref. 6) who used the Kirkwood-Bethe assumption (Ref. 7) for the solution of the problem. Figure 1 compares the result obtained by Gilmore with the exact solution of the compressible flow equations which have been obtained by numerical integration (Ref. 8). The numerical integration shows that the bubble wall velocity tends to infinity as $(R_o/R)^{0.785}$; Gilmore's analysis gives a velocity which increases like $(R_o/R)^{0.5}$, while the incompressible theory, as Eq. (15) shows, gives $(R_o/R)^{1.5}$. Since the Gilmore equation is an analytically expressed result and since it has such remarkable accuracy, an outline of its development may well be included here.

We continue to use the assumption of spherical symmetry in the compressible case as well as in the incompressible case. The velocity field is therefore derivable from a potential,

$$\vec{v} = -\nabla\varphi \quad ,$$

and the motion is irrotational so that the equation of motion is again

$$\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right] = -\nabla p + \left(\frac{4\mu}{3} + \zeta\right)\nabla(\nabla\cdot\vec{v}) \quad . \tag{16}$$

In Eq. (16) μ is the ordinary coefficient of viscosity and ζ a second coefficient of viscosity related to the bulk viscosity. The last term in this equation may be transformed by the equation of mass conservation

$$\nabla\cdot\vec{v} = -\frac{1}{\rho}\frac{D\rho}{Dt}$$

where $D/Dt = \partial/\partial t + (\vec{v}\cdot\nabla)$ is the particle derivative. It is evident that the last term in (16) disappears either if the viscosity vanishes or if the compressibility vanishes. In the development of an approximate theory we are supposing that the effect of viscosity is small and that the effect of compressibility is moderately small. It is reasonable to neglect the term in Eq. (16) which is a combination of these two small effects.* Equation (16) may now be integrated to give

$$-\frac{\partial\varphi}{\partial t} + \frac{v^2}{2} = -\int_{P_o}^{p}\frac{dp}{\rho} \quad . \tag{17}$$

In Eq. (17) we have supposed that the pressure, P_o, at infinity is constant and that the velocity field also is constant at infinity; we have also assumed that the liquid density, ρ, is a function of the pressure alone. When the flow is isentropic with no heat flow or viscous dissipation, there will be a unique relationship $\rho = \rho(p)$. Even when viscous effects and heat flow processes are present, their effect on the density is usually not important for liquids. It is a familiar fact that the coefficient of

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*It should be emphasized that Eq. (16) is based on the assumption that the viscosity coefficients are constants; actually in most liquids, the variation of viscosity with pressure is greater than the variation of density with pressure. Therefore the term neglected in Eq. (16) is smaller than other terms due to variable viscosity which have already been omitted from this equation.

thermal expansion of liquids is small. We recognize the integral on the right hand side of Eq. (17) as the enthalpy

$$h(p) = \int_{P_o}^{p} \frac{dp}{\rho} \quad .$$

Equation (17) may be written as

$$h + \frac{1}{2} v^2 = \frac{\partial \varphi}{\partial t} \quad . \tag{17'}$$

Further, since we are concerned with a spherically symmetric flow, the only component of the fluid velocity $\vec{v}$ is in the radial direction. The vector momentum equation has only a radial component which may be written in the form

$$\frac{Dv}{Dt} = - \frac{\partial h}{\partial r} \quad , \tag{18}$$

with the omission of the viscosity-compressibility interaction term. The continuity equation is

$$\frac{\partial v}{\partial r} + \frac{2v}{r} = - \frac{1}{c^2} \frac{Dh}{Dt} \quad , \tag{19}$$

where $c^2 = dp/d\rho$ with c the local value of the sound velocity.

The Kirkwood-Bethe approximation has so far not been deduced in a rigorous, straightforward way, but it may be made plausible by the following argument. We are interested in a collapsing spherical cavity so that the liquid flow consists entirely of outgoing velocity and pressure waves. As long as the collapse continues, we shall have only these expansion waves and no discontinuities, or shocks, will appear in the liquid flow field. If the flow velocities are small compared with the sound velocity and if the sound velocity everywhere has essentially its constant value at infinity, the velocity field would be described by the familiar potential for diverging spherical sound waves:

$$\varphi(r, t) = \frac{1}{r} f(t - r/c_\infty) \quad , \tag{20}$$

where r is the distance from the center of the bubble. With this potential, Eq. (17') gives

$$r(h+v^2/2) = f'(t-r/c_\infty) \quad . \tag{21}$$

Equations (20) and (21) show that both $r\varphi$ and $r(h+v^2/2)$ have an outward wave propagation with the velocity c_∞. This approximation may be called the "quasi-acoustic" approximation since the term in v^2 is ordinarily neglected as being unimportant compared with h. For the present problem we have large transport velocities and we should keep this term. To go beyond the quasi-acoustic approximation, it is plausible to assume that either $r\varphi$ or $r(h+v^2/2)$ is propagated outward with a variable velocity $(c+v)$ where c is the appropriate local velocity of sound and v is the local transport velocity. These two possibilities are only approximately equivalent. The Kirkwood-Bethe approximation consists in taking the second possibility which may be expressed quantitatively as

$$\frac{\partial}{\partial t}\left[r(h+v^2/2)\right] = -(c+v)\frac{\partial}{\partial r}\left[r(h+v^2/2)\right] \quad .$$

This equation may also be written in the form

$$\frac{D}{Dt}\left[r(h+v^2/2)\right] + c\,\frac{\partial}{\partial r}\left[r(h+v^2/2)\right] = 0 \quad ,$$

or, in expanded form

$$r\,\frac{Dh}{Dt} + rv\,\frac{Dv}{Dt} + (c+v)(h+v^2/2) + rc\,\frac{\partial h}{\partial r} + rcv\,\frac{\partial v}{\partial r} = 0 \quad . \tag{22}$$

Now, if derivatives with respect to r are eliminated from Eq. (22) by means of Eqs. (18) and (19), one obtains

$$r\,\frac{Dh}{Dt}\left(1 - \frac{v}{c}\right) + ch\left(1 + \frac{v}{c}\right) - rc\,\frac{Dv}{Dt}\left(1 - \frac{v}{c}\right) - \frac{3}{2}\,cv^2\left(1 - \frac{v}{3c}\right) = 0 \quad . \tag{23}$$

Equation (23) is an equation which describes the liquid flow field in the assumed approximation. The bubble wall is a "particle" path and the particle derivatives in Eq. (23) may be used directly to determine the motion of the bubble wall. The values of quantities like r, h, etc. at the bubble wall will be denoted by capital letters R, H, etc. Then Eq. (23) gives

$$R\ddot{R}\left(1 - \frac{\dot{R}}{C}\right) + \frac{3}{2}\,\dot{R}^2\left(1 - \frac{\dot{R}}{3C}\right) = H\left(1 + \frac{\dot{R}}{C}\right) + \frac{R\dot{H}}{C}\left(1 - \frac{\dot{R}}{C}\right) . \tag{24}$$

When the equation of state is given H and C are known as functions of p, and the problem is completely specified with the boundary condition (8). Equation (24) is Gilmore's equation for the collapse of a spherical bubble in a compressible liquid. The equation may be rewritten to give a differential equation for the bubble wall velocity $U = \dot{R}$ in terms of R:

$$RU \frac{dU}{dR}\left(1 - \frac{U}{C}\right) + \frac{3}{2} U^2\left(1 - \frac{U}{3C}\right) = H\left(1 + \frac{U}{C}\right) + \frac{RU}{C}\frac{dH}{dR}\left(1 - \frac{U}{C}\right) . \quad (25)$$

The particular initial condition which we have taken corresponds to the bubble at rest and a uniform pressure P_o and zero velocity throughout the liquid. At $t = 0$ the pressure at the bubble wall is abruptly reduced to the value $p(R) < P_o$. This "release" of the bubble gives a jump in velocity of the bubble wall from zero to U_o where U_o is determined from Eq. (25). The important terms in this equation for the determination of this U_o come from the derivatives dU/dR, and dH/dR so that Eq. (25) gives

$$RU(1-U/C)dU = (RU/C)(1-U/C)dH \quad .$$

we then find

$$U_o = \int_o^H \frac{dH}{C} \approx \frac{p(R)-P_o}{\rho_\infty c_\infty} \quad .$$

Equation (25) may be integrated once analytically for the case for which surface tension and viscosity are omitted in the boundary condition (10) and, in addition, the pressure within the bubble is assumed to remain constant. In this case we get

$$\ln \frac{R}{R_o} = -2\int_{U_o}^{U} \frac{U(U - C)dU}{U^3 - 3CU^2 + 2HU + 2HC} \quad . \quad (27)$$

In most bubble collapse situations we have

$$|H| \ll C^2 \quad ,$$

an inequality which is certainly valid for water. If this inequality is used, Eq. (27) gives

$$\left(\frac{R_o}{R}\right)^3 = \left(1 - \frac{U}{3C}\right)^4\left(1 + \frac{3\rho_\infty U^2}{2(P_o - p_v)}\right) \quad . \quad (28)$$

When viscous and surface tension effects are included and when the pressure, p_v, within the bubble is not taken to be constant, a numerical integration of Eq. (25) must be carried out. The case shown in Fig. 1 is for an empty bubble with $P_o - p_v = P_o = P_\infty$. The Kirkwood-Bethe approximation is seen to be quite accurate as far as

$$R/R_o \sim 5 \times 10^{-3} \quad \text{and for} \quad |U|/C \lesssim 5 \quad .$$

We have already observed that for large collapse velocities the pressure of the vapor within the bubble will rise above the equilibrium value. As the pressure within the bubble rises, the collapse motion will eventually be arrested and the bubble will rebound and begin to grow. This rebound problem has been considered (Ref. 8) with the exact compressible flow equations for the liquid, but the conditions within the bubble have been approximated with the assumption that the bubble contains a small amount of permanent gas. In this calculation the main interest was in an evaluation of the magnitude of the sharp pressure pulses which are radiated upon bubble rebound. It was found that peak pressures, in typical cases, would be of the order of 1,000 atm and that the pressure field attenuates with distance as $1/r$ as does an acoustic pulse. It is clear that the actual conditions within a collapsing cavitation bubble are more complicated than those considered in Ref. 8, but one may expect that those calculations give a resonable estimate of the pressure pulses radiated.

The complications which appear in the collapse of a cavitation bubble do not cause great difficulties until the radius of the bubble becomes a small fraction of the initial radius, and some successful analyses of the problem are available. As complete a discussion of the collapse of a boiling bubble is not yet available. This problem appears after a vapor bubble has grown in a superheated liquid, $p_v > P_o$, and is transported into a region of cooled liquid where $p_v < P_o$. We may suppose that the bubble collapses from rest where the radius has the maximum value R_o. As the radius decreases, the heat of condensation must be conducted away from the bubble wall into the liquid. The complication in this problem comes from the fact that the heat flow problem

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial r} = D\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) \quad ; \quad v = R^2\dot{R}/r^2 \quad ; \tag{29}$$

can not be considered without reference to the momentum equation

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left[p_v(T) - P_o - 2\sigma/R\right] \tag{30}$$

as was done in the asymptotic phase of boiling bubble growth. The collapse of a boiling bubble has the same kind of difficulties as are encountered in the initial phases of boiling bubble growth. If we consider the complete trajectory $R(t)$ of a bubble growing first in a superheated liquid and then passing into a subcooled liquid so that the growth is arrested and collapse takes place, we may show that in the neighborhood of the maximum radius

$$\frac{R}{R_o} = 1 - \gamma(t - t_o)^2 \tag{31}$$

where γ is some constant. It is also evident that the general trajectory will not be symmetrical about the maximum bubble radius. An approximate solution has been attempted by Dougherty and Rubin (Ref. 9), but a detailed analysis is not yet available.

An aspect of the collapse problem which remains to be discussed is the stability of the spherical shape. Even in the absence of gravity it may be shown that distortions from the spherical shape appear fairly early in the collapse history. The deformations become marked when $R/R_o \sim 0.1$ and continue to grow as R decreases. This instability sets in before many of the other complications, such as arise from viscosity or compressibility, become important. A problem which should be considered further is the effect of these instabilities on the last stages of the collapse.

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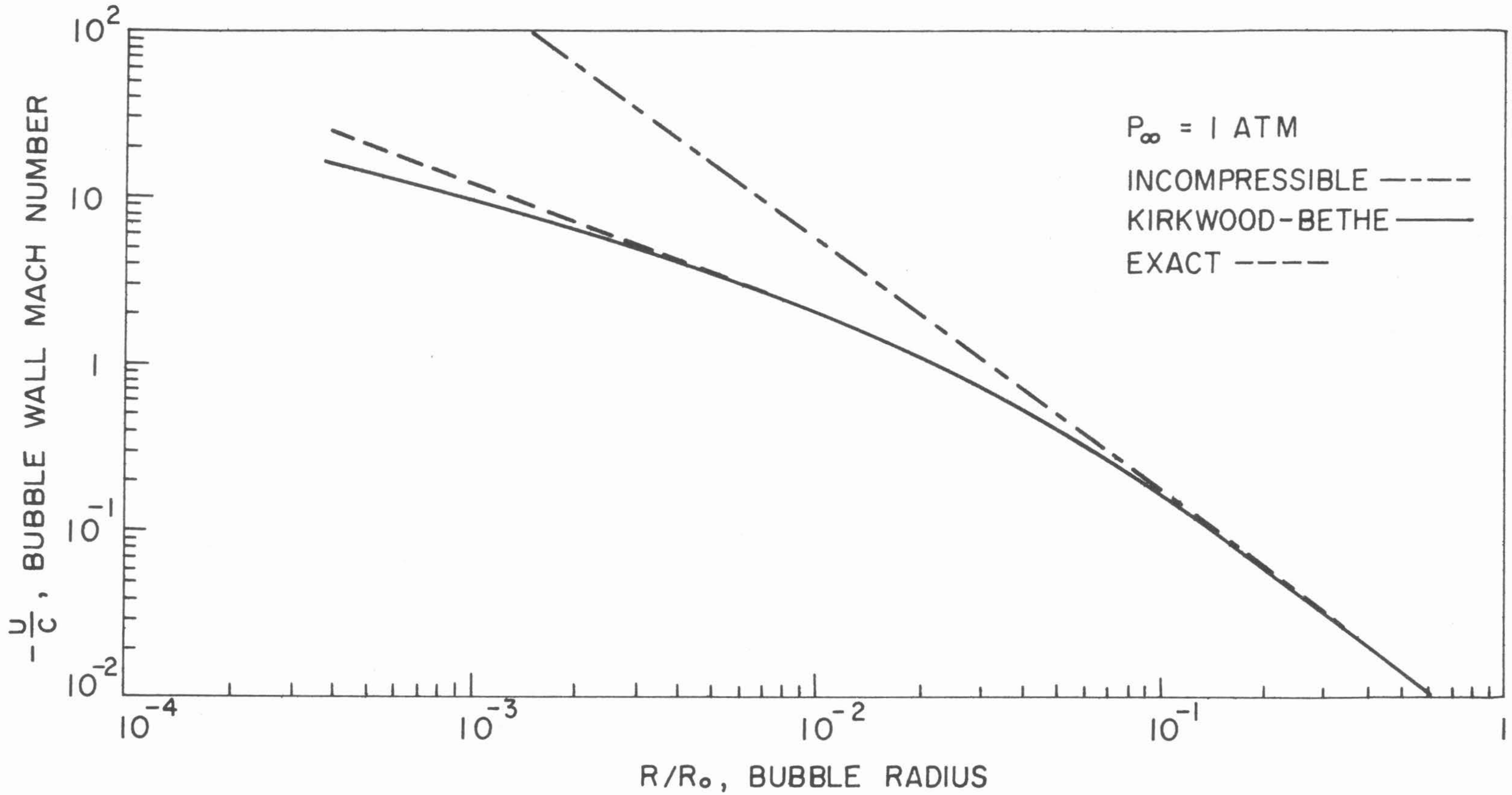

$-\frac{U}{c}$, BUBBLE WALL MACH NUMBER
10^2
10
1
10^{-1}
10^{-2}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
R/R₀, BUBBLE RADIUS
P_∞ = 1 ATM
INCOMPRESSIBLE
KIRKWOOD-BETHE
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FIG. 1

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An analysis is given of the theory of the growth and collapse of vapor cavities for supercooled liquids and for superheated liquids. For the problem of the collapse of a vapor cavity particular attention is given to the theoretical effects of compressibility.

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